4.

$$2xy' + 2xy = x^3 e^{-x}$$

dividing through by $2x$

$$y' + y = \frac{x^3 e^{-x}}{2x}$$

$$y' + y = \frac{1}{2} x^2 e^{-x}$$

This corresponds to

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I(x) = e^{\int P(x)\,dx} = e^{\int 1\,(dx)} = e^x$$

$$y(x) = \frac{1}{e^x}\left[\int e^x e^{-x} \cdot \frac{1}{2} x^2 \cdot \right.$$

$$= \frac{1}{e^x} \cdot \int \frac{1}{2} x^2 \cdot$$

$$= \frac{1}{e^x} \frac{x^3}{6} + c \cdot$$

$$y(x) = \frac{1}{e^x}\left[\frac{x^3}{6} + c\right]$$

10.

$$y'' + 5y' + 6y = (t^2 + 2t + 1)e^{2t}$$

General soln.

$$m^2 + 5m + 6 = 0.$$

$$m^2 + 3m + 2m + 6 = 0.$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0.$$

$$(m+2)(m+3)$$

$$m_1 = -2, \quad m_2 = -3$$

General soln $y = Ae^{-3x} + Be^{-2x}$

$$F(t) = (t^2 + 2t + 1)e^{2t}$$

$$F_p = (At^2 + Bt + C)e^{2t}$$

$$F_p = (A_2t^2 + A_1t + A_0)e^{2t}.$$